YEAR 12 MATHEMATICS SPECIALIST

SEMESTER ONE 2019

TEST 3: Vectors

WESLEY COLLEGE

By daring & by doing

Name: _____

Monday 20$^{\text{th}}$ May

Time: 50 minutes

Total marks: $\dfrac{}{20} + \dfrac{}{30} = \dfrac{}{50}$

Calculator free section – maximum 15 minutes

1. [9 marks – 2, 3, 2, 1 and 1]

    For the vectors $p = 2i + j - 2k$ and $q = 2i - 2j + k$

    (a) show that p is perpendicular to q

    (b) calculate, in simplest form, a vector r that is perpendicular to both p and q

    There is a plane that includes both p and q and passes through $A(1, -2, 3)$. Write an equation for this plane in:

    (c) normal form

    (d) Cartesian form

    (e) vector form $r = a + \lambda b + \mu c$

2. [5 marks]

   Calculate the point of intersection of the planes defined by the simultaneous system

   $$\begin{cases} x + 2y + 3z = 10 \\ 2x - y - z = 3 \\ x + y + 4z = 9 \end{cases}$$

3. [6 marks – 2, 1, 1 and 2]

   When Gaussian elimination, using elementary row operations, was applied to a system of equations in variables x, y and z, this augmented echelon matrix resulted:

   $$P = \begin{bmatrix} k^2 - 4 & 0 & 0 & | & k + 2 \\ 1 & -2 & 0 & | & 3 \\ 1 & 3 & -2 & | & 8 \end{bmatrix}$$

   For which value(s) of k will:

   (a) the system have no solutions

   (b) there be an infinite number of solutions

   (c) $x = 1$?

   Hence:

   (d) Evaluate y and z when $x = 1$

4. [6 marks – 2, 2, and 2]

Two points A and B have position vectors $OA = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$ and $OB = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$.
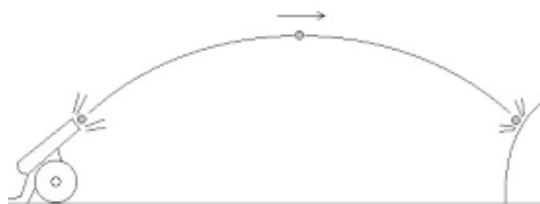
Determine:

(a) the exact area of ΔOAB

(b) (measure or size of) $\angle AOB$

(c) two different vectors of length $\sqrt{3}$ that are perpendicular to both OA and OB

5. [9 marks –1, 2, 1, 1, 2 and 2]

When Rodney, the human cannonball in the Kleenheat TV advertisement, is fired from his cannon, his position vector, at time t seconds $t \geq 0$, is defined, in metres, by

$$r(t) = 10t\,i + (10 + 5t - 4.9t^2)\,j$$

Determine:

(a) his velocity vector $v(t)$

(b) his initial angle of elevation

(c) his acceleration

(d) when he reaches the high point of his trajectory

(e) his height (above ground level) when he hits his target 18 m (horizontally) from the cannon

(f) the length of his trajectory

6. [8 marks – 5, 1 and 2]

(a) Calculate the minimum distance from the point $A(-3,7,-8)$ to the plane
$3x-4y+5z=23$

(b) Determine the equation of the sphere, centred at $A(-3,7,-8)$ that is tangential to
$3x-4y+5z=23$

(c) Identify another plane that is tangential to the sphere in (b) with the same minimum distance from $A(-3,7,-8)$.

7. [7 marks –2, 2, 1, 1 and 1]

An ice-skater follows a cyclic path with her velocity given by $v(t) = 6\cos 2t\,i - 4\sin t\,j$

(a) Write an expression for her position $r(t)$ given that she started at $r(0) = 4j$

(b) Sketch $r(t)$, clearly indicating the direction of travel
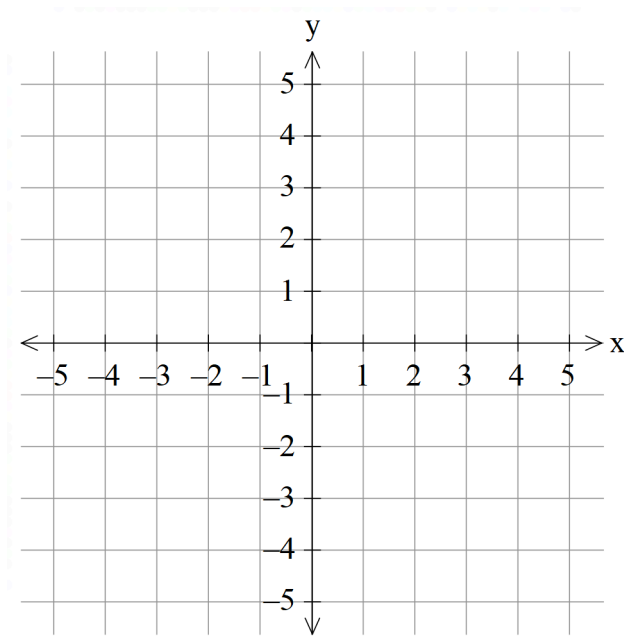
(c) How long does she take to complete one circuit?

Identify point(s) on the circuit when the acceleration is:

(d) 0

(e) directly toward the origin